Reusable Formal Verification of DAG-based Consensus Protocols (In TLA+)

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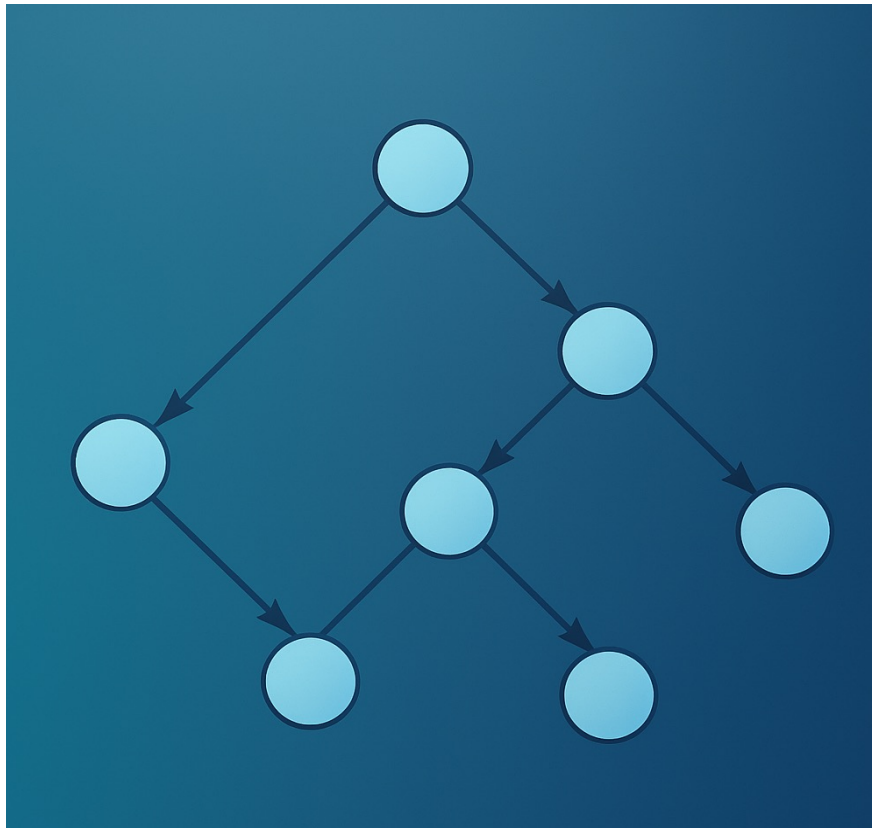

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DAG-based Consensus Protocols

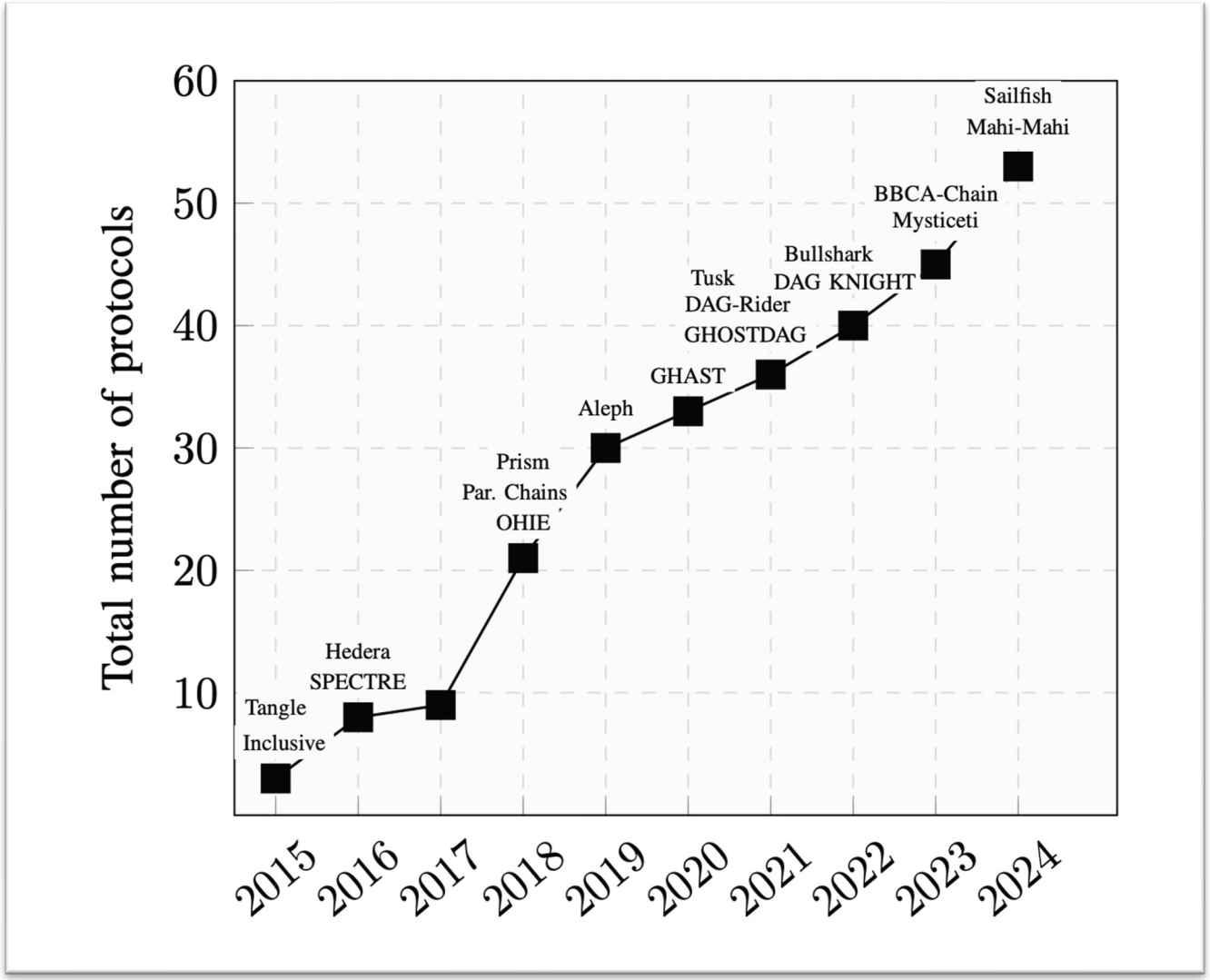

Figure Source:
Raikwar *et al.*, *SoK: DAG-based Consensus Protocols*, IEEE ICBC 2024.

DAG-based Consensus Protocols

Improvements over Classical Blockchains:

- ❑ High performance
- ❑ Low communication complexity
- ❑ Byzantine fault tolerance

Used in Many Modern Blockchains:

Verification of DAG-based Consensus Protocols

- **Testing:** Often misses corner-case interleavings.
- **Model Checking:** Does not scale to real-world instances.
- **Parameterized Model Checking:** Not yet expressive enough.

Theorem Proving - the only viable option for rigorous verification!

Verification of DAG-based Consensus Protocols

- Proving correctness of each DAG-based consensus protocol **from scratch is infeasible**.

Verification of DAG-based Consensus Protocols

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- We demonstrate that DAG-based consensus protocols are amenable to **practical, reusable,** and **compositional** formal methods.

Verification of DAG-based Consensus Protocols

✅ **Safety Verified Specifications in TLA+:**

- **DAG-Rider**
- **Cordial Miners**
- **Hashgraph**
- **Eventually Synchronous BullShark**
- **A variant of Aleph**

With proof effort reduced by almost 50%

DAG-based Consensus Protocols Solve the Byzantine Atomic Broadcast Problem

- N processes; some may be Byzantine-faulty.
- Each process can propose and output blocks.
- All correct processes eventually output same set of blocks and in the same order (**Agreement, Total order**).
- No correct process outputs same block more than once (**Integrity**).
- A block proposed by a correct process is eventually output by all correct processes (**Validity**).

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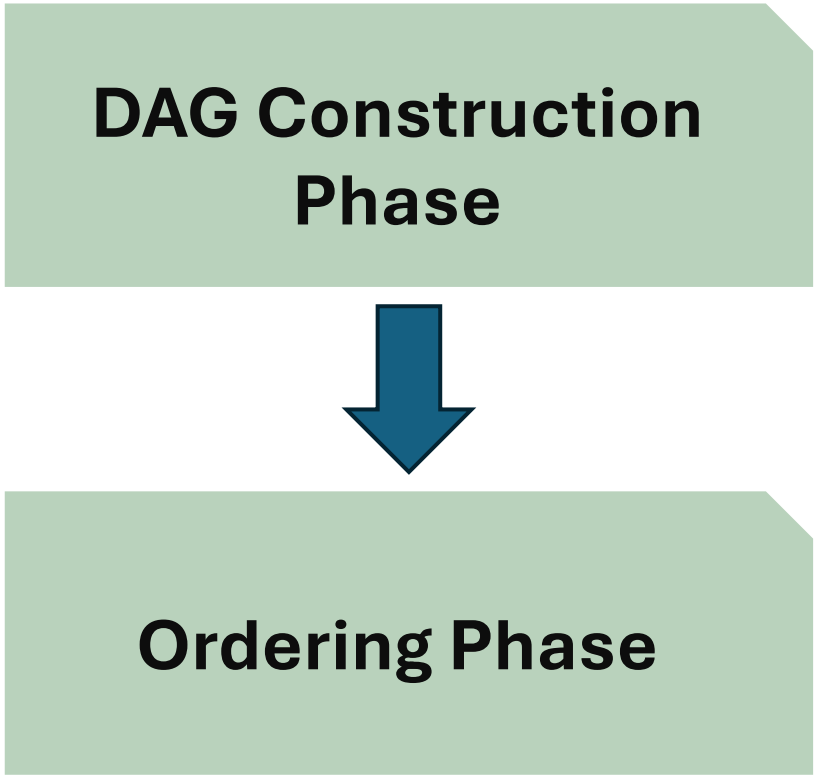
DAG-based Consensus Protocols Solve the Byzantine Atomic Broadcast Problem in Two Phases

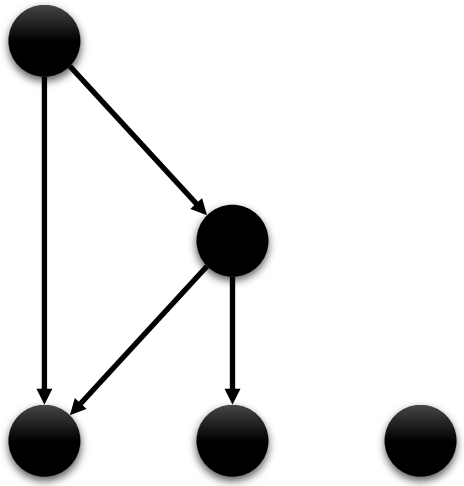


Processes **communicate** their blocks and **build a DAG** of exchanged blocks.

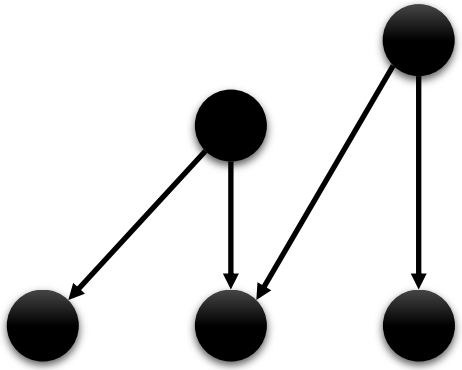
Processes use their locally constructed DAGs to determine the **total order** of the blocks.

DAG Construction Phase

Processes Communicate Their Blocks and Build a DAG of Exchanged Blocks



P1



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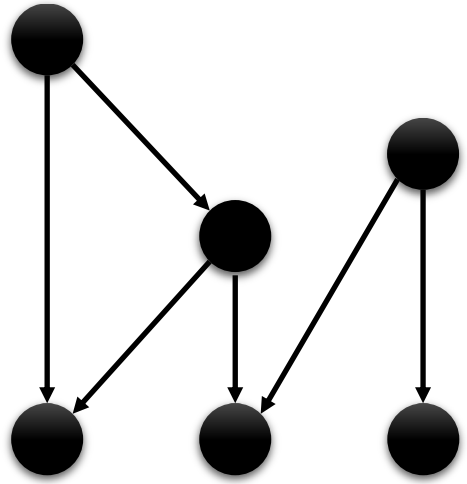
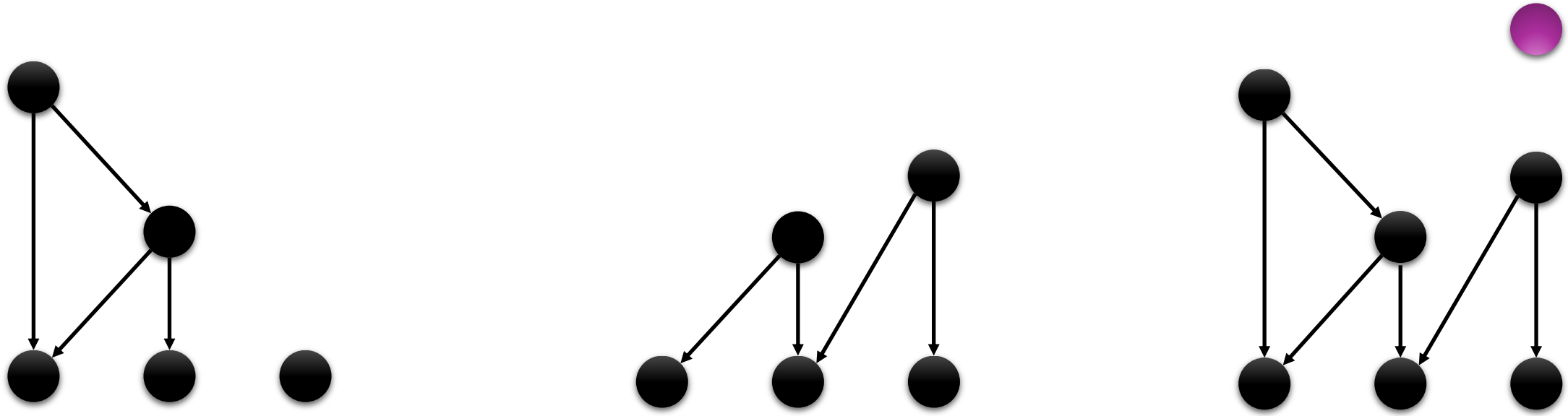


P3

DAG Construction Phase

Processes Communicate Their Blocks and Build a DAG of Exchanged Blocks

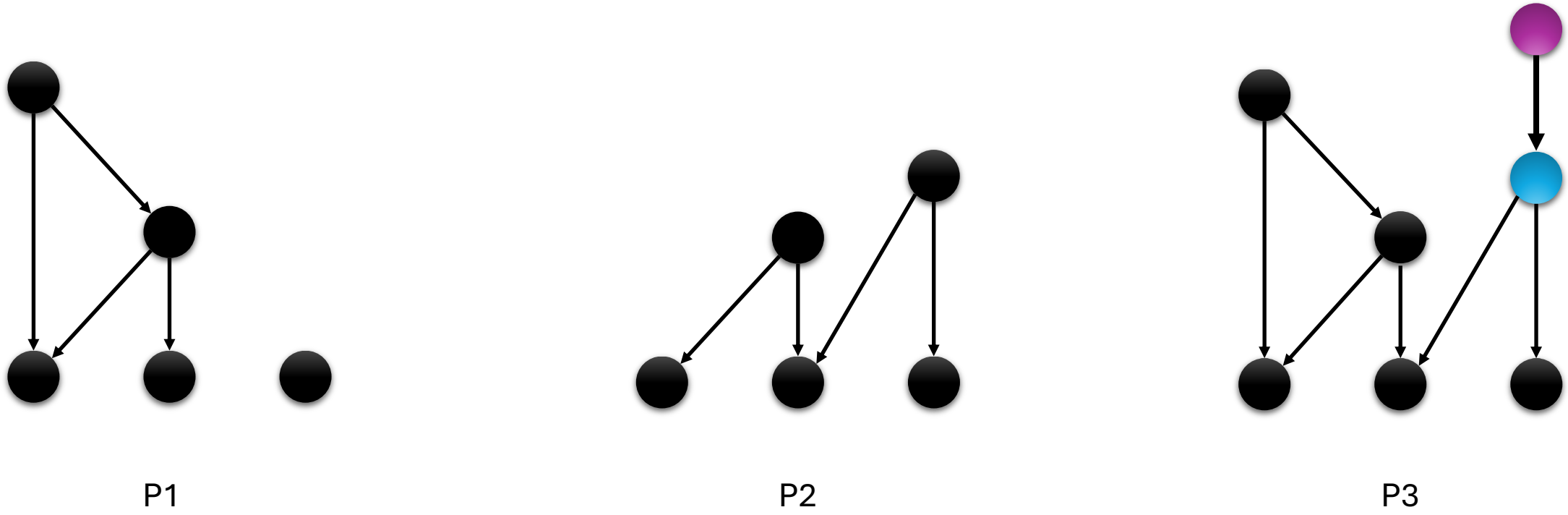


P1 P2 P3

A process creates new blocks in the form of vertices.

DAG Construction Phase

Processes Communicate Their Blocks and Build a DAG of Exchanged Blocks



A process creates a new vertex by referencing it to its last vertex and other vertices in its local DAG.

DAG Construction Phase

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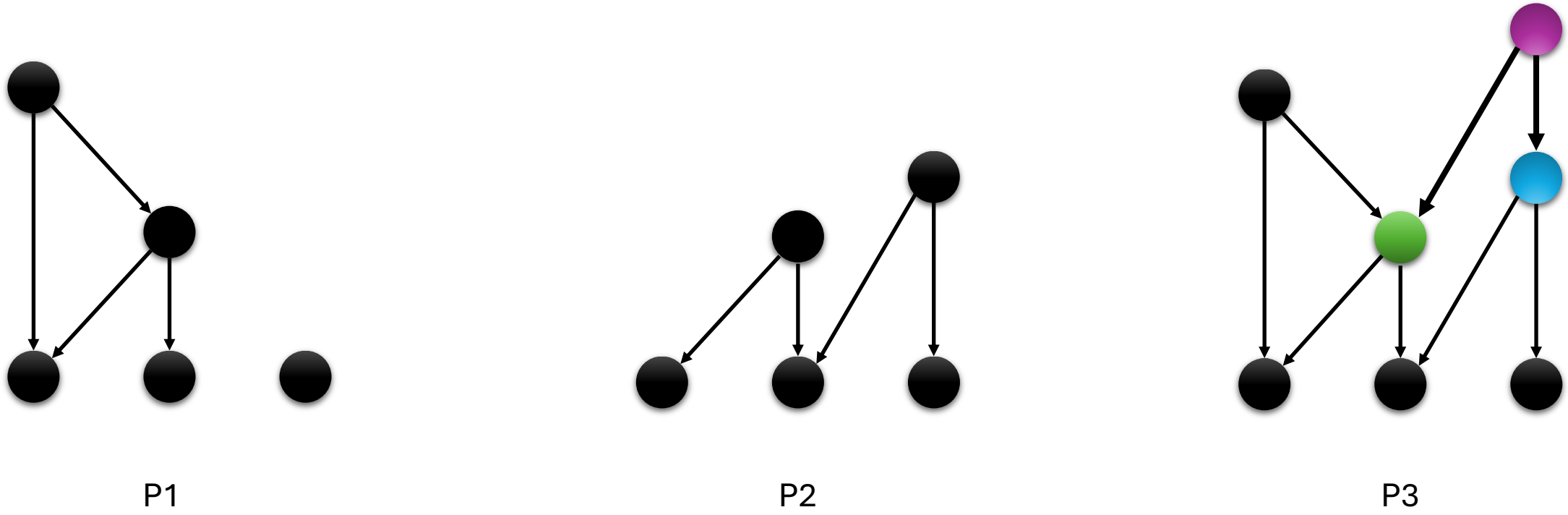

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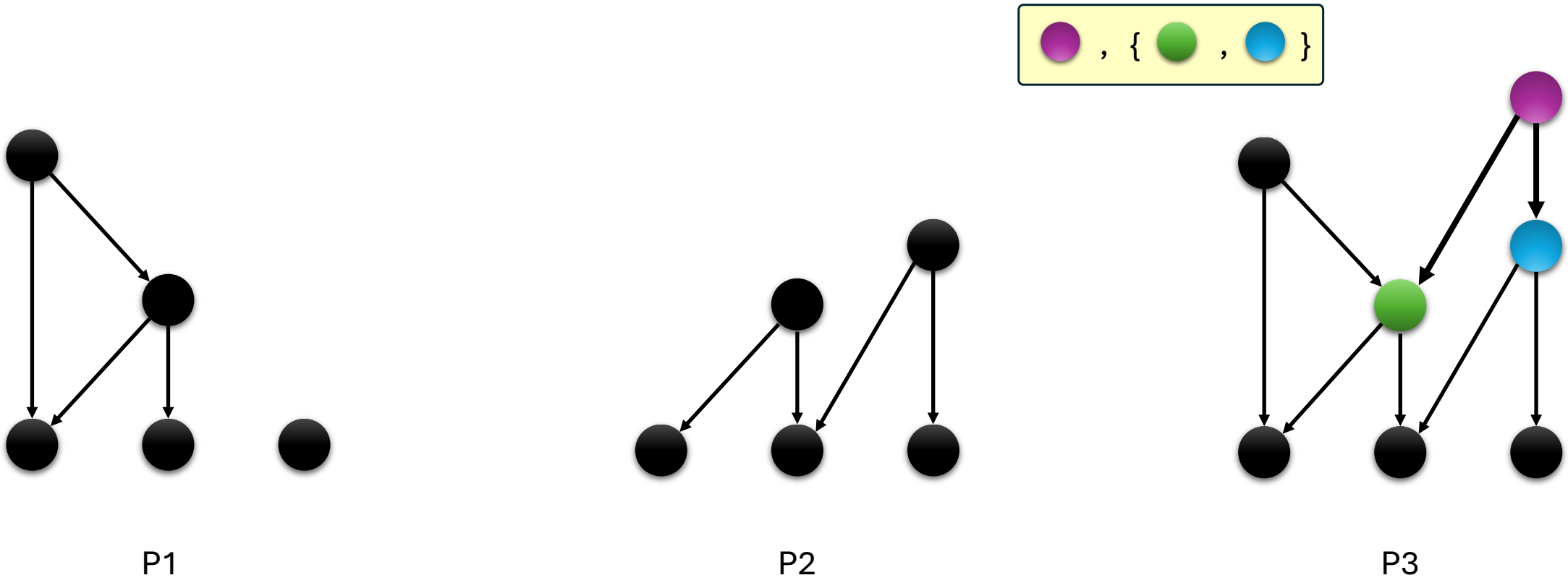
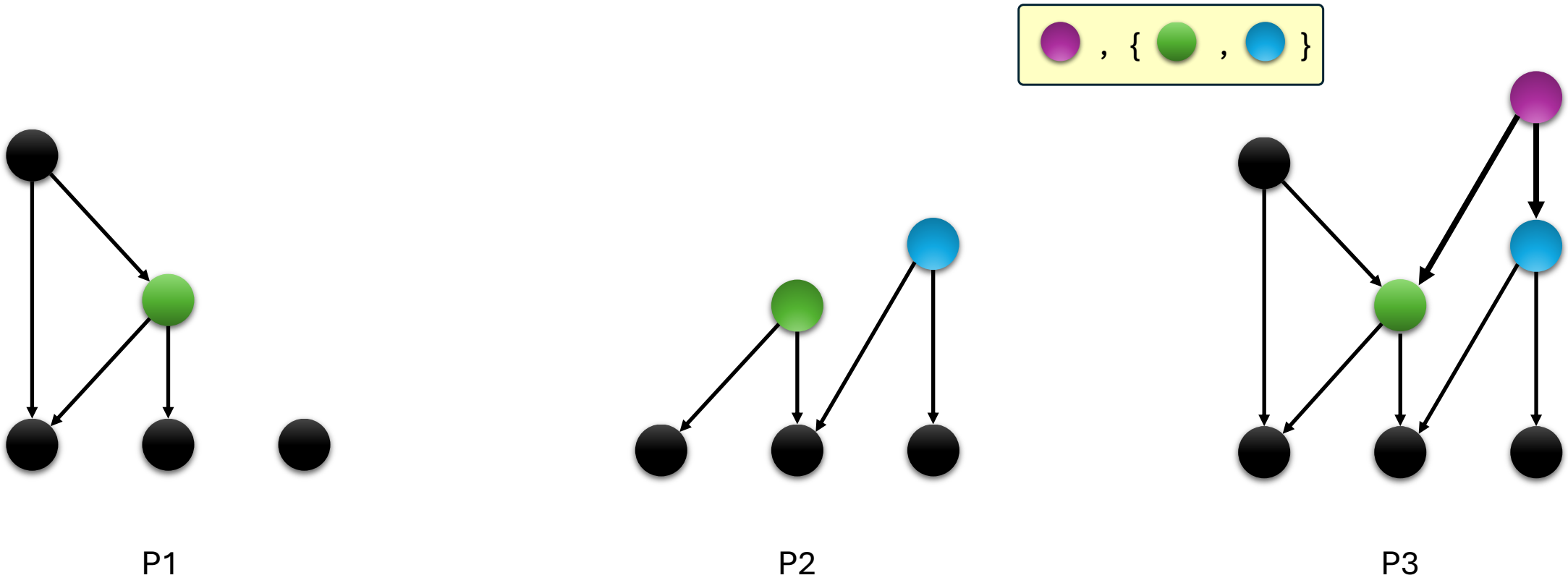


A process communicates newly created vertices along with their references to other processes.

DAG Construction Phase

Processes Communicate Their Blocks and Build a DAG of Exchanged Blocks



When a process receives a vertex from another processes, it checks whether it has all its references in its local DAG.

DAG Construction Phase

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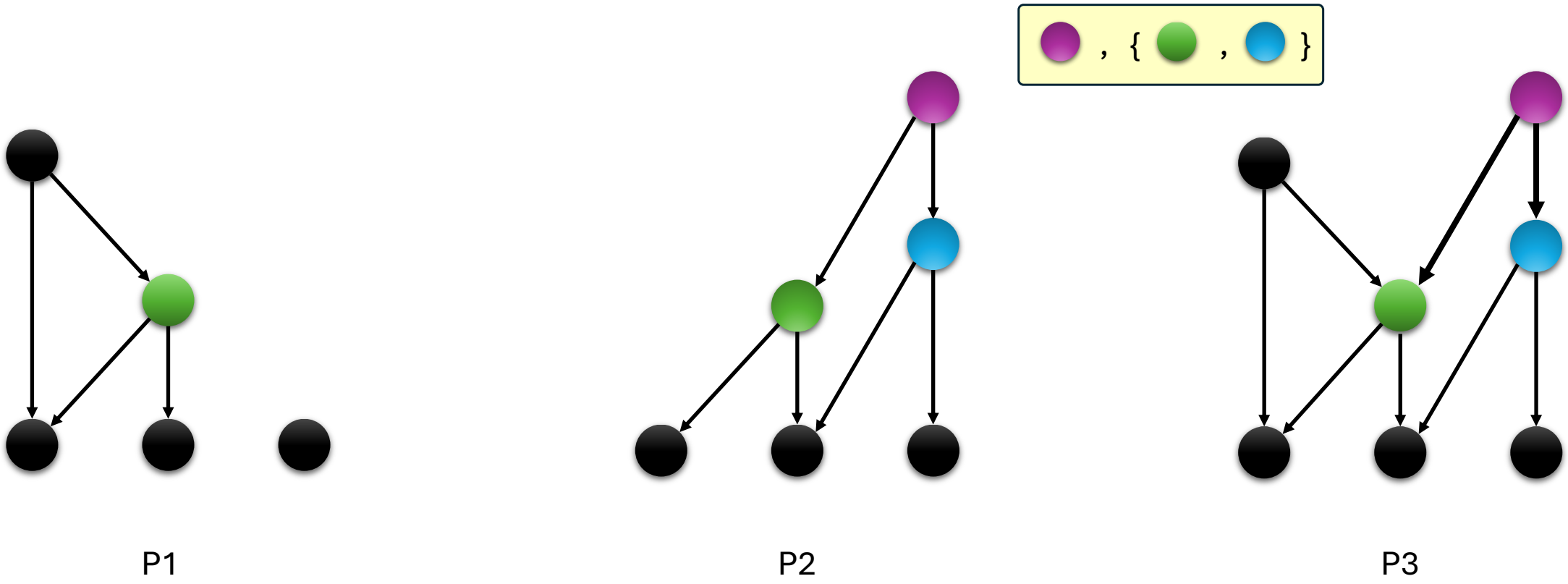
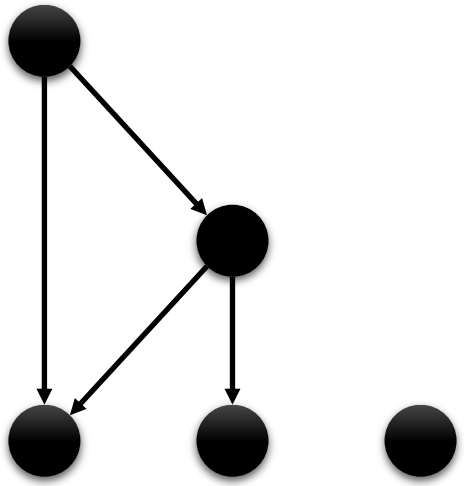


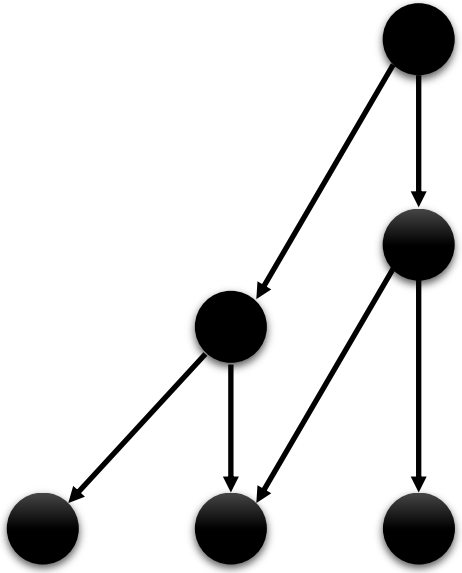
If the references exist in the process's local DAG, it adds the vertex; otherwise, it stores it in a buffer.

DAG Construction Phase

Processes Communicate Their Blocks and Build a DAG of Exchanged Blocks



P1



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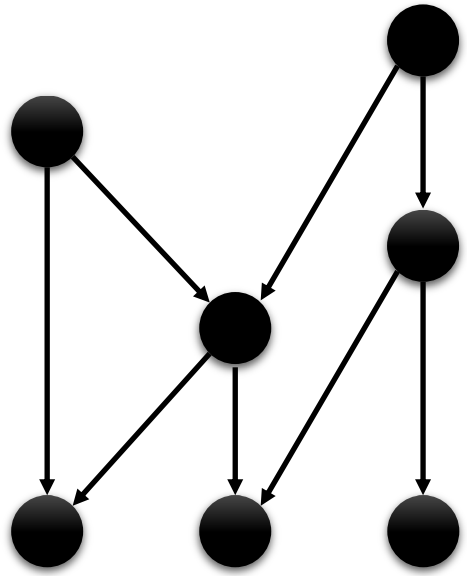
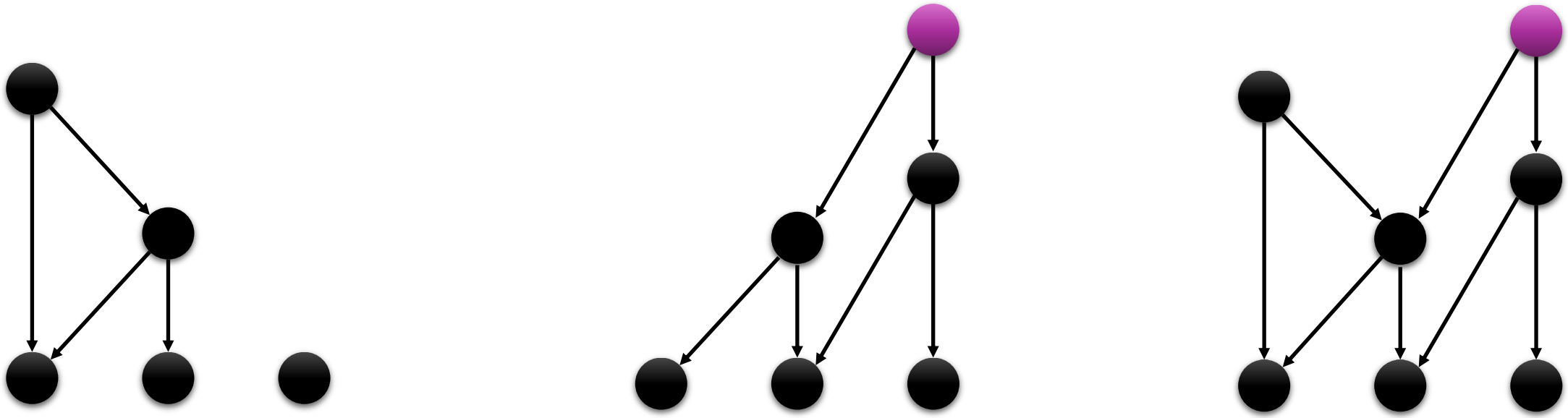


P3

DAG Construction Phase

DAG Construction Ensures the Consistent Causal History Property

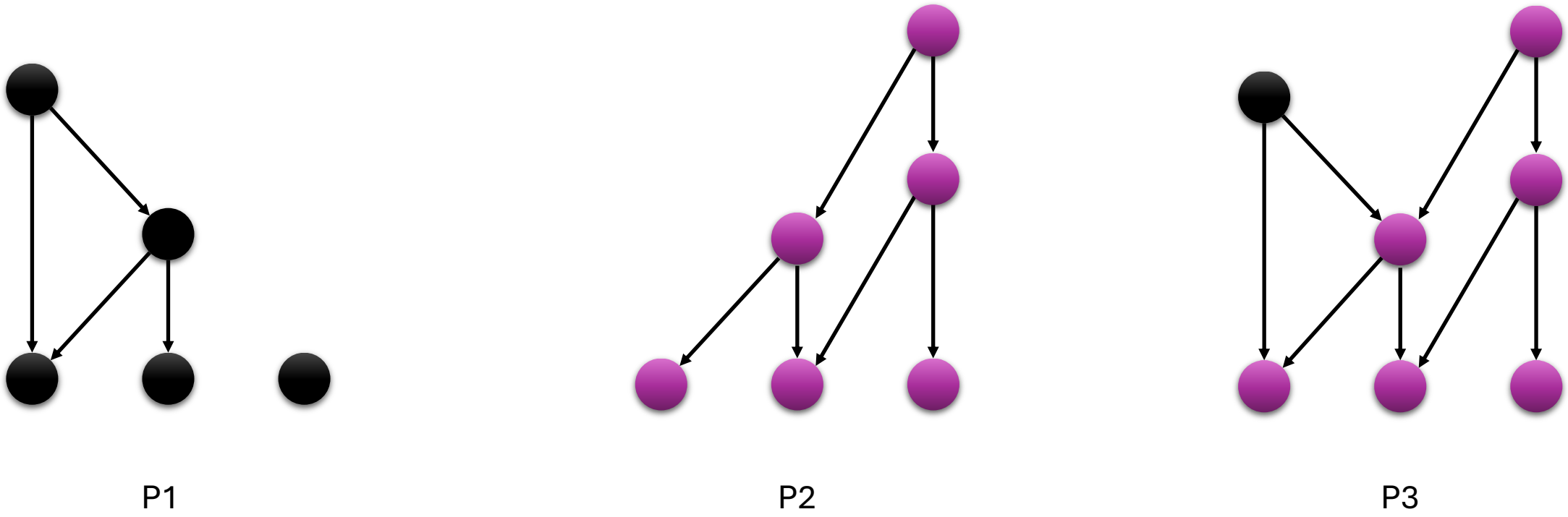


P1 P2 P3

If a vertex is present in the local DAG of two correct processes, then their causal histories are the same.

DAG Construction Phase

DAG Construction Ensures Consistent Causal History Property



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Protocol	DAG Construction Phase	
	Comm-unication	DAG-Type
DAG-Rider		
Cordial Miners		
ES BullShark		
Aleph		
Hashgraph		

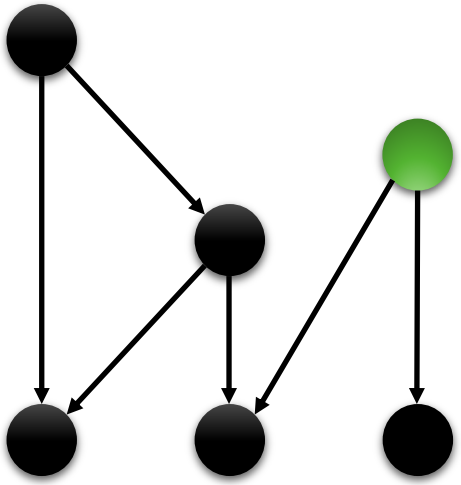
Protocol	DAG Construction Phase	
	Comm-unication	DAG-Type
DAG-Rider	Reliable (RB)	
Cordial Miners	Unreliable	
ES BullShark	Reliable (RB)	
Aleph	Reliable (RB)	
Hashgraph	Unreliable	

RB: Reliable Broadcast

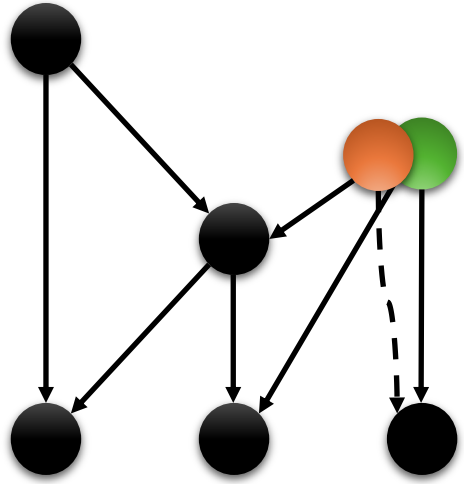
DAG Construction Phase

Reliable vs. Unreliable Communication

Reliable Communication
(reliable broadcast)



Unreliable Communication
(plain broadcast/ gossip)



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Aleph	Reliable (RB)	
Hashgraph	Unreliable	

RB: Reliable Broadcast

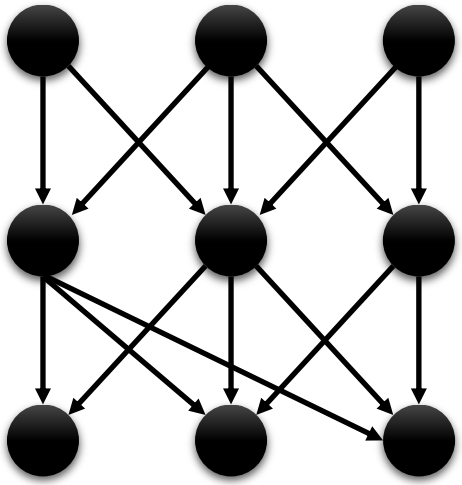
Protocol	DAG Construction Phase	
	Comm-unication	DAG-Type
DAG-Rider	Reliable (RB)	Structured
Cordial Miners	Unreliable	Structured
ES BullShark	Reliable (RB)	Structured
Aleph	Reliable (RB)	Structured
Hashgraph	Unreliable	Unstructured

RB: Reliable Broadcast

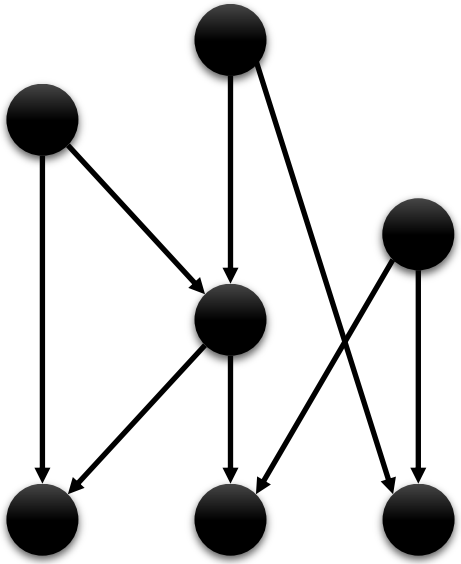
DAG Construction Phase

Structured vs. Unstructured DAG Construction

Structured DAG Construction
(round-driven)



Unstructured DAG Construction
(event-driven)



Protocol	DAG Construction Phase	
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Protocol	DAG Construction Phase		Ordering Phase
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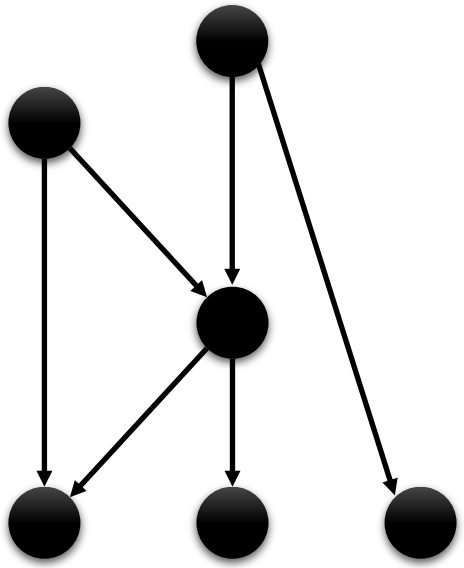
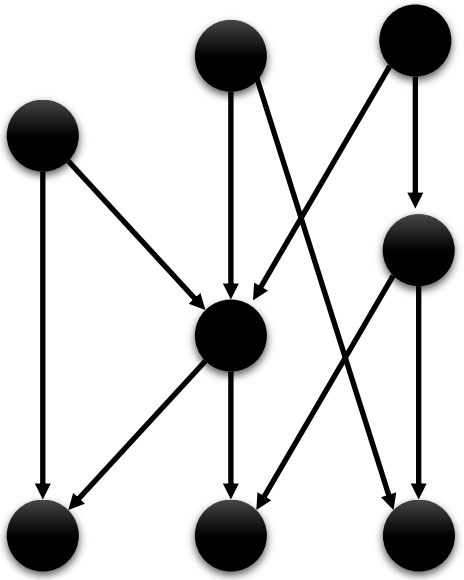
Ordering Phase

Processes Use Their Local DAGs to Determine the Total Order of the Vertices



P1



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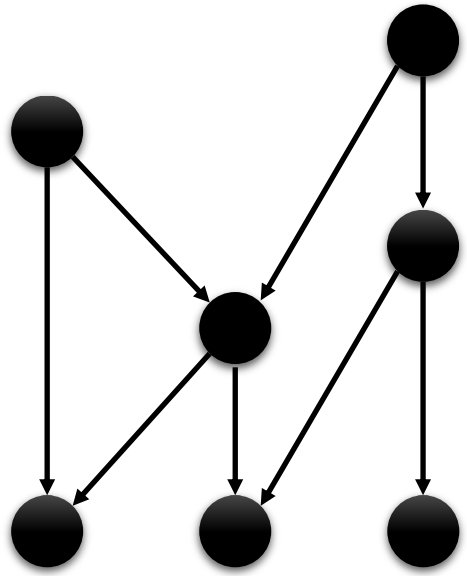
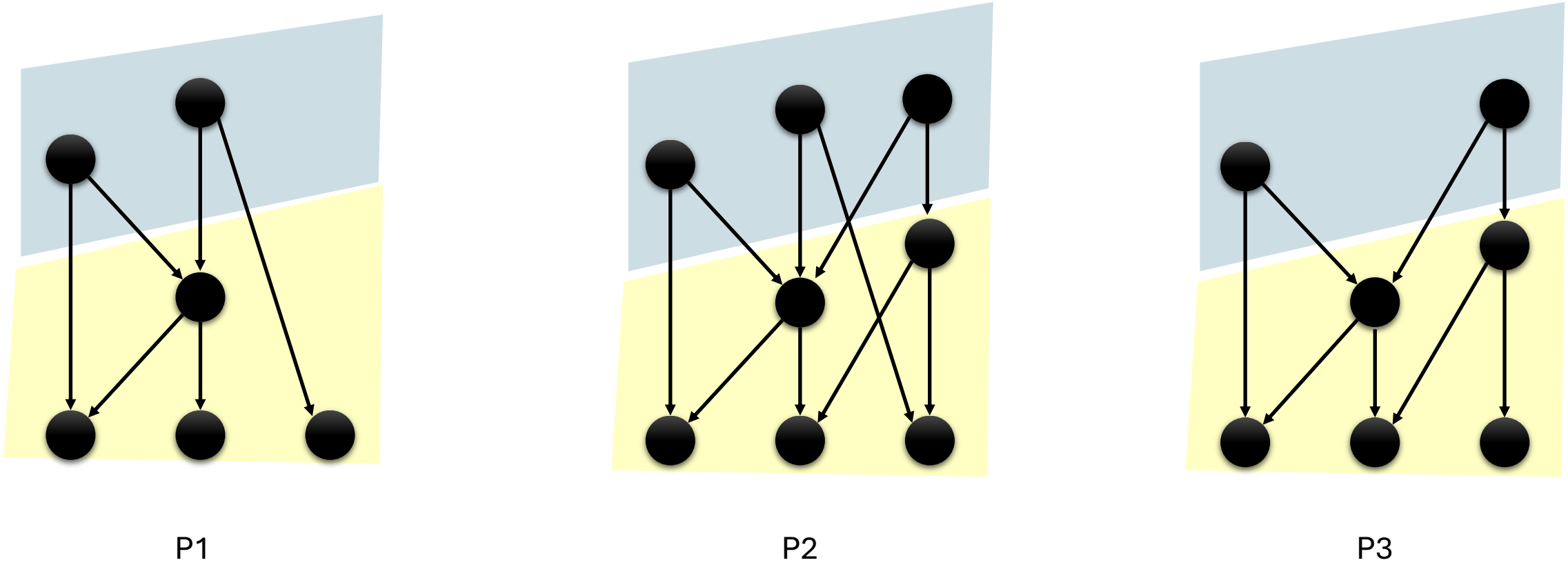


P3

Ordering Phase

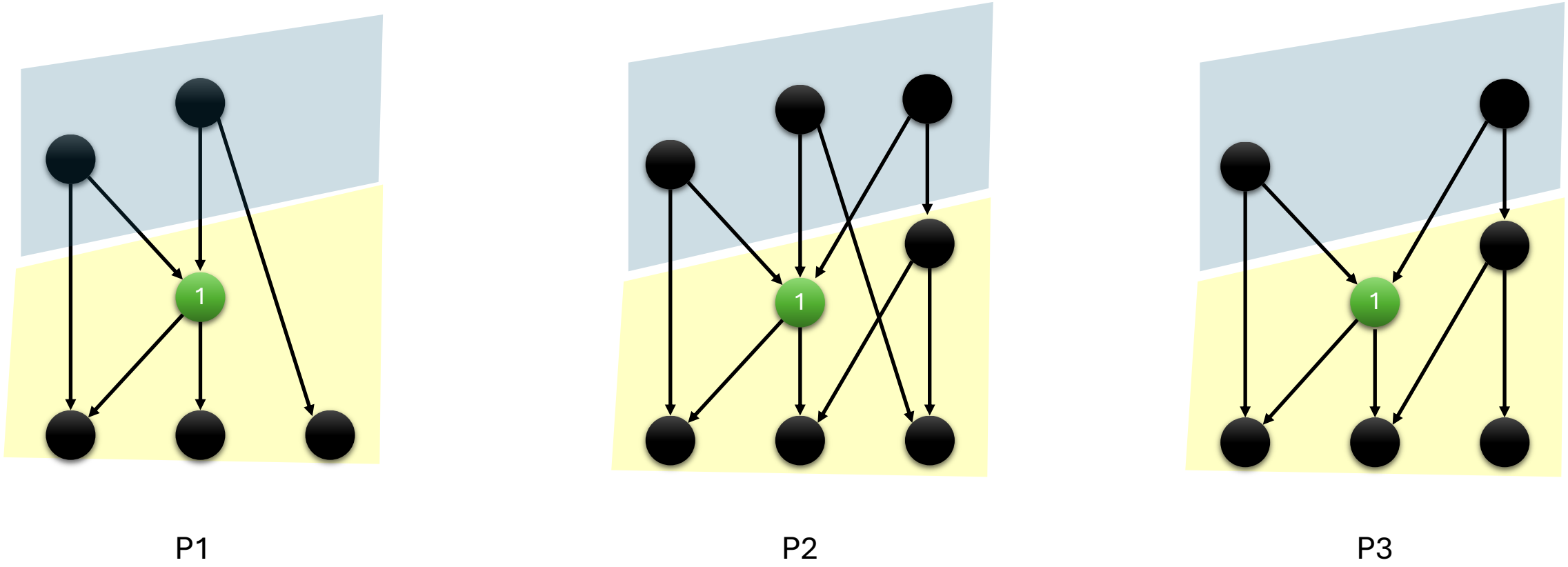
Frame Construction: Partitioning DAGs into Sequential Frames



P1 P2 P3

Ordering Phase

Anchor Selection: Agree on a Vertex per Frame



P1 P2 P3

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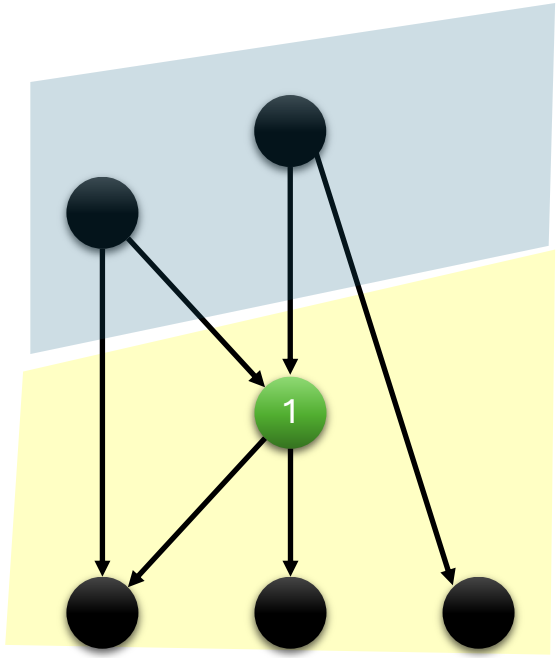

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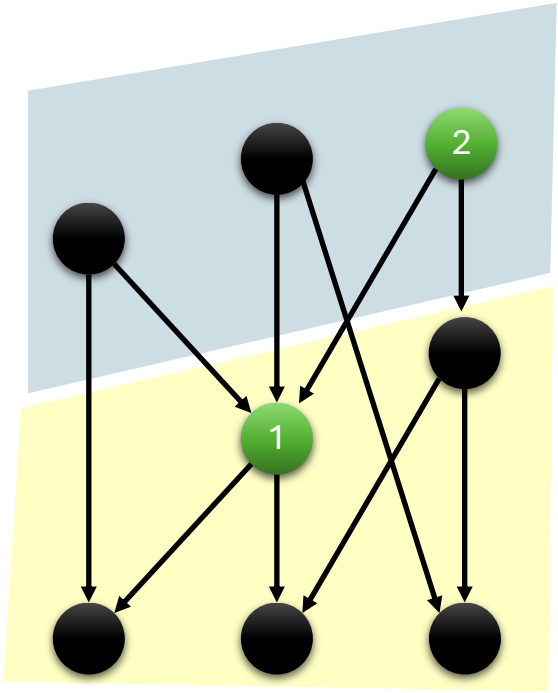

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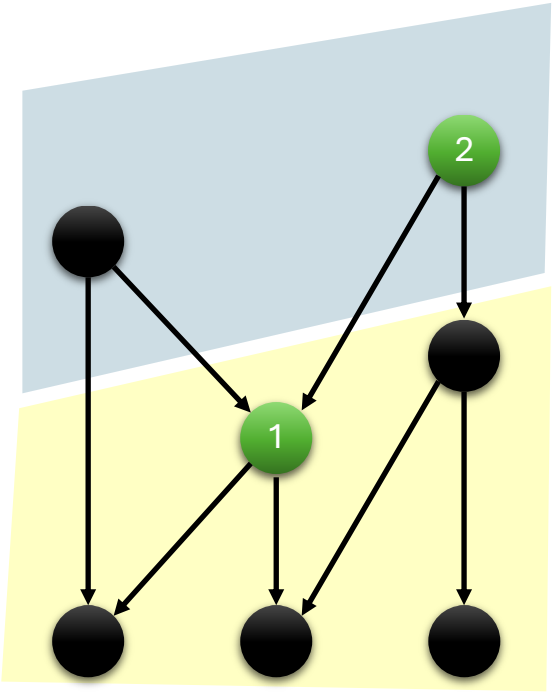
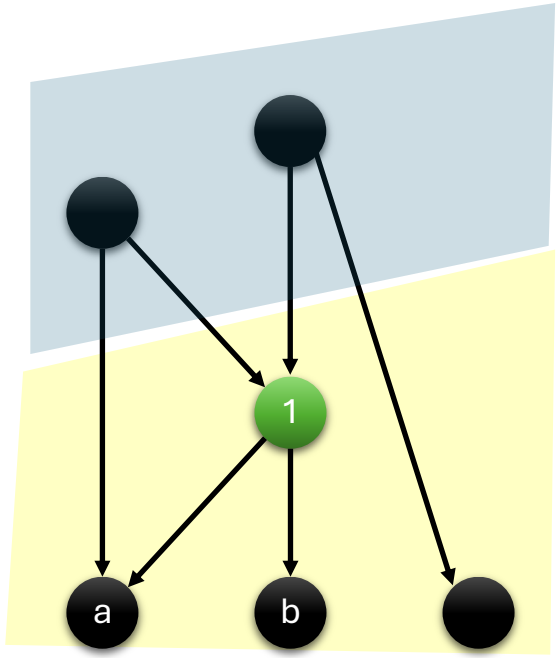


P3

Ordering Phase

Total Ordering: Using the Causal Histories of Anchor Vertices



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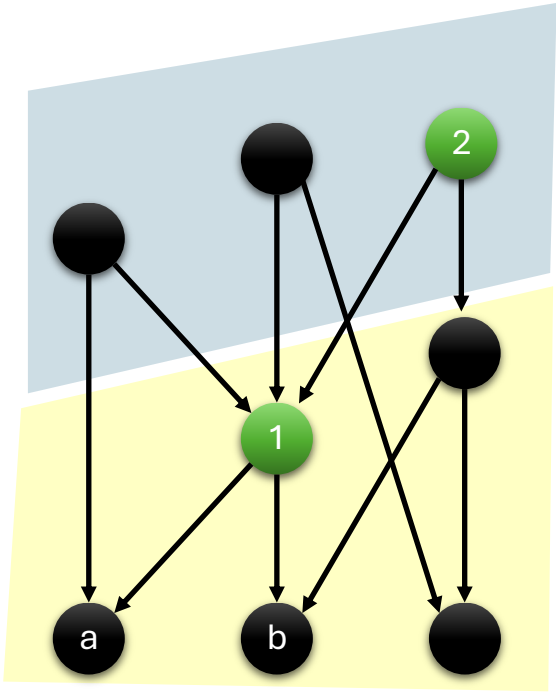

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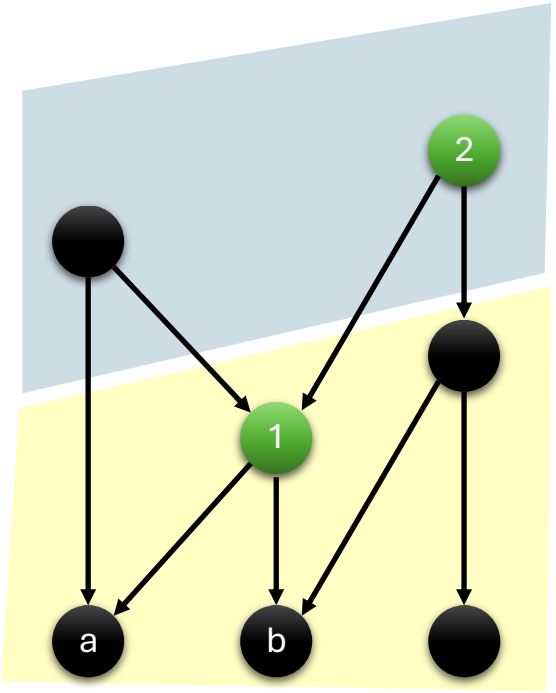

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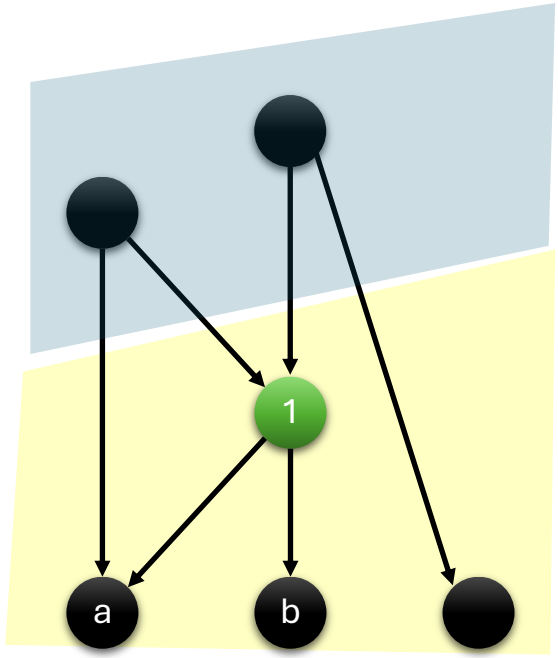

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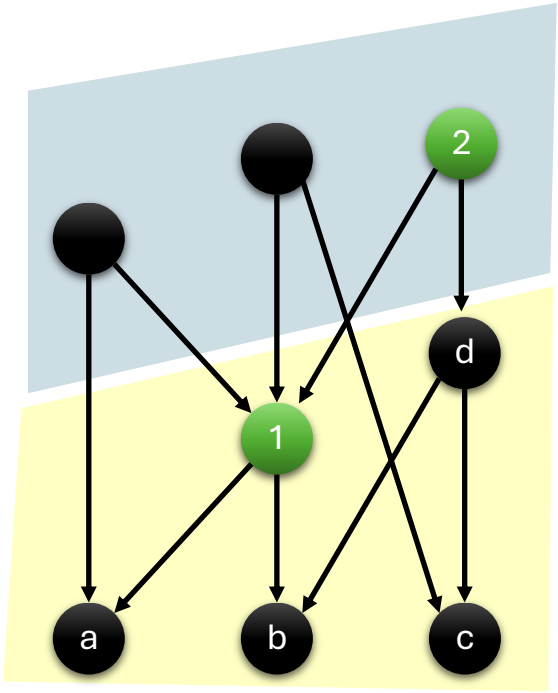

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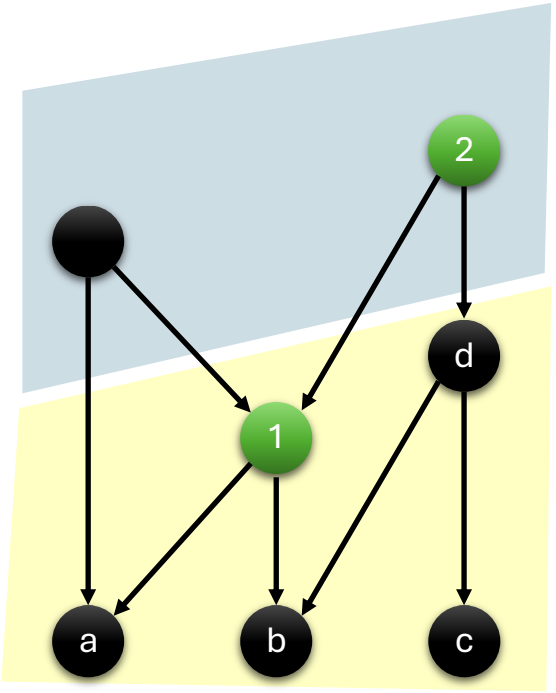

P3

Protocol	DAG Construction Phase		Ordering Phase
	Comm-unication	DAG-Type	
DAG-Rider	Reliable (RB)	Structured	
Cordial Miners	Unreliable	Structured	
ES BullShark	Reliable (RB)	Structured	
Aleph	Reliable (RB)	Structured	
Hashgraph	Unreliable	Unstructured	

RB: Reliable Broadcast

Protocol	DAG Construction Phase		Ordering Phase
	Comm-unication	DAG-Type	Anchor Agreement
DAG-Rider	Reliable (RB)	Structured	GPC-based
Cordial Miners	Unreliable	Structured	GPC-based
ES BullShark	Reliable (RB)	Structured	GPC-style (deterministic)
Aleph	Reliable (RB)	Structured	Virtual Voting based
Hashgraph	Unreliable	Unstructured	Virtual Voting based

RB: Reliable Broadcast, GPC: Global Perfect Coin

Protocol	DAG Construction Phase		Ordering Phase	
	Comm-unication	DAG-Type	Anchor Agreement	Fork Handling
DAG-Rider	Reliable (RB)	Structured	GPC-based	-
Cordial Miners	Unreliable	Structured	GPC-based	Required
ES BullShark	Reliable (RB)	Structured	GPC-style (deterministic)	-
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Hashgraph	Unreliable	Unstructured	Virtual Voting based	Required

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Aleph	Reliable (RB)	Structured	-	-	Virtual Voting based
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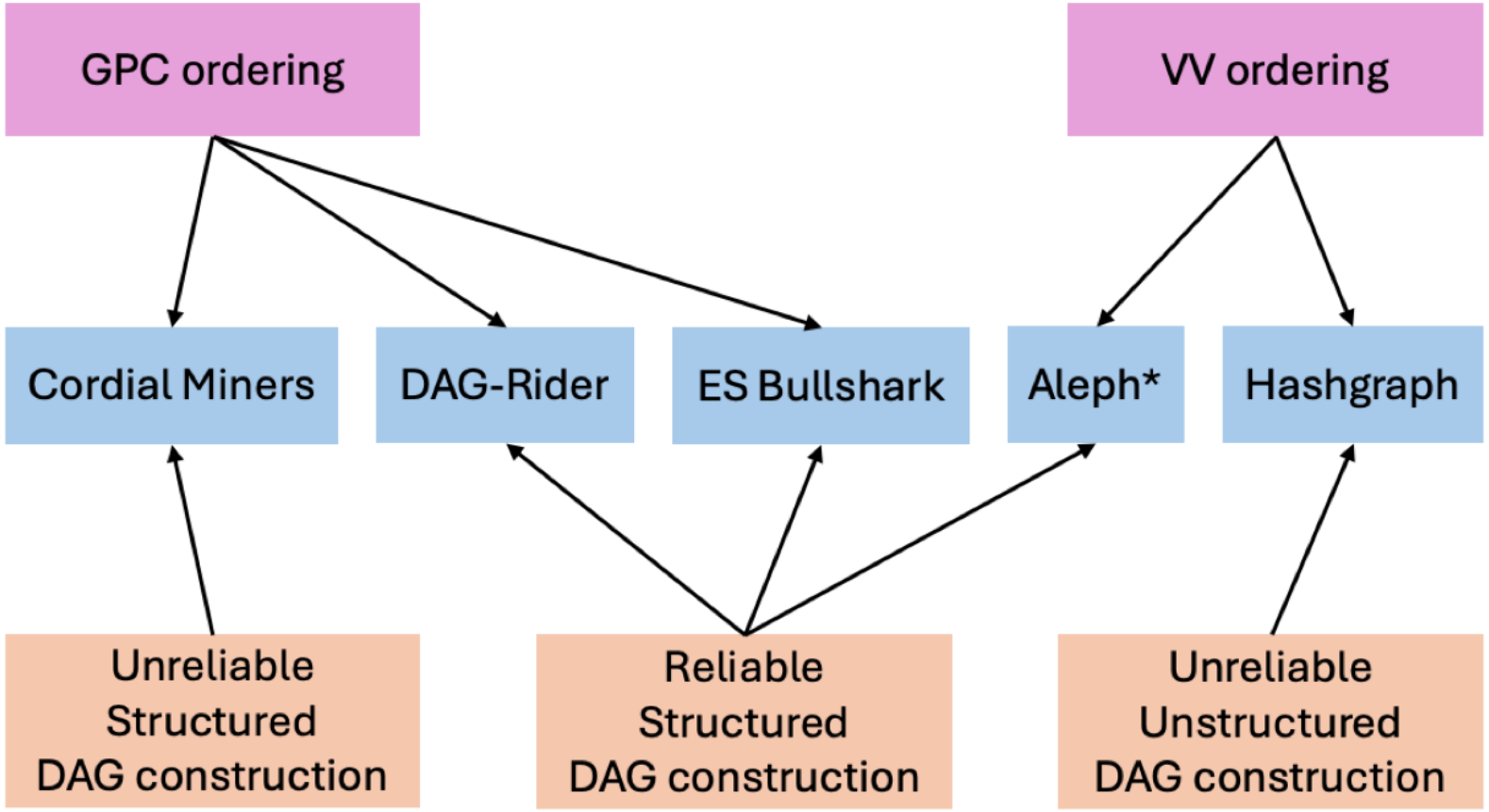
Building Blocks and Their Use in Verifying Five DAG-based Consensus Protocols



GPC: Global Perfect Coin, VV : Virtual Voting

Specifications and Proofs in TLA+ (Checked with TLAPS)

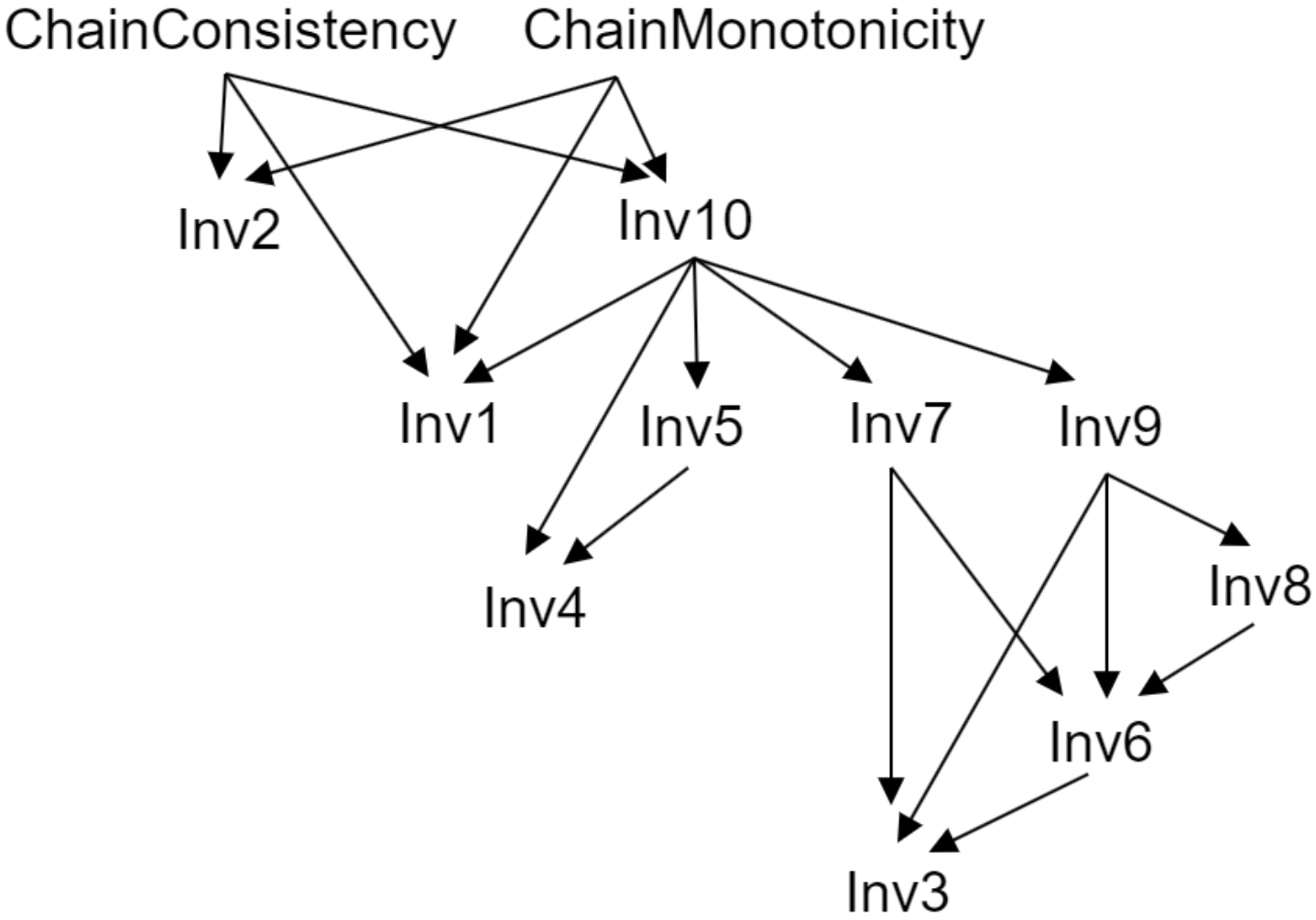
Why TLA+?

- ❑ **Produces a specification closer to the implementation.**
- ❑ **Produces execution traces.**
- ❑ **Supports interface refinement.**

Specifications and Proofs in TLA+ (Checked with TLAPS)

- ❑ **Specify safety properties as safety invariants.**
- ❑ **In TLA+ Invariants are proved by induction.**
- ❑ **The hardest challenge: identify relevant inductive invariants that imply the safety invariants.**

Verifying Safety Invariants of GPC Ordering



Proof of an Invariant in TLA+

```
LEMMA IndInv5Lem == Spec => []IndInv5
 <1>1 Init => IndInv5
      BY DEF Init, InitBlocksToPropose, InitBroadcastNetwork, InitBroadcastRecord, InitBuffer, InitDag, InitRound, IndInv5
 <1>2 ASSUME [Next]_vars, StateType, StateType', IndInv5, IndInv2
      PROVE IndInv5'
      <2>1 ASSUME NEW p \in ProcessorSet, NEW b \in BlockSet, ProposeTn(p, b)
           PROVE IndInv5'
           BY VertexSetDefPlt, <2>1, <1>2 DEF IndInv5, ProposeTn
      <2>2 ASSUME NEW p \in ProcessorSet, NextRoundTn(p)
           PROVE IndInv5'
           <3>1 ASSUME NEW r \in RoundSet, NEW v \in VertexSet, Broadcast(p, r, v)
                PROVE IndInv5'
                <4>1 CASE broadcastRecord[p][r] = FALSE
                     <5>1 broadcastNetwork'["History"] = broadcastNetwork["History"] \cup {[sender |-> p, inRound |-> r, vertex |-> v]}
                          BY <3>1, <2>2, <1>2, <4>1 DEF StateType, BlocksToProposeType, BroadcastNetworkType, BroadcastRecordType, BufferType, DagType, RoundType, Broadcast
                     <5>2 broadcastRecord' = [broadcastRecord EXCEPT ![p][r] = TRUE]
                          BY <3>1, <2>2, <1>2, <4>1 DEF StateType, BlocksToProposeType, BroadcastNetworkType, BroadcastRecordType, BufferType, DagType, RoundType, Broadcast
                     <5>3 ASSUME NEW m \in broadcastNetwork'["History"], NEW o \in broadcastNetwork'["History"], m.sender = o.sender, m.inRound = o.inRound
                          PROVE m = o
                          <6>1 CASE m \in broadcastNetwork["History"] /\ o = [sender |-> p, inRound |-> r, vertex |-> v]
                               <7>1 broadcastRecord[m.sender][m.inRound] = TRUE
                                    BY <6>1, <1>2 DEF IndInv2
                               <7> QED BY <4>1, <7>1, <6>1, <5>3
                          <6>2 CASE o \in broadcastNetwork["History"] /\ m = [sender |-> p, inRound |-> r, vertex |-> v]
                               <7>1 broadcastRecord[o.sender][o.inRound] = TRUE
                                    BY <6>2, <1>2 DEF IndInv2
                               <7> QED BY <4>1, <7>1, <6>2, <5>3
                          <6>3 CASE m \in broadcastNetwork["History"] /\ o \in broadcastNetwork["History"]
                               BY <6>3, <5>3, <1>2 DEF IndInv5
                          <6> QED BY <6>1, <6>2, <6>3, <5>3, <5>1
                     <5> QED BY <1>2, <5>3 DEF IndInv5
                <4>2 CASE broadcastRecord[p][r] = TRUE
                     <5>1 UNCHANGED <<broadcastNetwork, broadcastRecord>>
                          BY <4>2, <2>2, <3>1 DEF Broadcast
                     <5> QED BY <5>1, <1>2 DEF IndInv5
                <4> QED BY <4>1, <4>2, <3>1, <2>2, <1>2 DEF StateType, BlocksToProposeType, BroadcastNetworkType, BroadcastRecordType, BufferType, DagType, RoundType
           <3> QED BY VertexSetDefPlt, <2>2, <1>2, CreateVertexTypePlt, <3>1 DEF IndInv5, NextRoundTn, Broadcast
      <2>3 ASSUME NEW p \in ProcessorSet, NEW r \in RoundSet, NEW q \in ProcessorSet, NEW v \in VertexSet, p# q, ReceiveVertexTn(p, q, r, v)
           PROVE IndInv5'
           <3>1 broadcastNetwork'["History"] =  broadcastNetwork["History"]
                <4>1 p # "History"
                     BY <2>3, ProcSetAsm DEF ProcessorSet
                <4> QED BY <4>1, <2>3, <1>2 DEF StateType, BlocksToProposeType, BroadcastNetworkType, BroadcastRecordType, BufferType, DagType, RoundType, ReceiveVertexTn
           <3> QED BY <3>1, VertexSetDefPlt, <2>3, <1>2 DEF IndInv5, ReceiveVertexTn
      <2>4 ASSUME NEW p \in ProcessorSet, NEW v \in VertexSet, AddVertexTn(p, v)
           PROVE IndInv5'
           BY VertexSetDefPlt, <2>4, <1>2 DEF IndInv5, AddVertexTn
      <2>5 CASE UNCHANGED vars
           BY VertexSetDefPlt, <2>5, <1>2 DEF IndInv5, vars
      <2> QED BY <1>2, <2>1, <2>2, <2>3, <2>4, <2>5 DEF Next
 <1> QED BY <1>1, <1>2, TypeLem, IndInv2Lem, PTL DEF Spec
```

Evaluation

Metric \ Phase	Reliable structured	Unreliable structured	Unreliable unstructured	GPC Ordering	VV Ordering
Size of spec. (# loc)	403	160	230	272	136
Number of invariants	6	6	7	10	18
Size of proof (# loc)	460	594	554	822	2120
Max level of proof tree nodes	10	9	8	9	13
Max degree of proof tree nodes	7	8	7	7	11
# obligations in TLAPS	633	895	665	1302	3316
Time to check by TLAPS (s)	49	68	74	125	651

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- Extracted **reusable building blocks,** reducing verification effort by ~50%.

Potential Impact on Verifying Other DAG-based Protocols:

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Thank You

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